

A Potential New Test for Unseen Matter
in **the** Outer **Solar** System

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Abstract

The recent development of trapped-ion frequency standards, which offer high stability over long time periods, provides us with a potential new method for detecting unseen matter in the outer solar system. A distribution of matter or a planetary body could produce a measurable gravitational redshift of the radio signal received from a spacecraft equipped with an ultrastable frequency standard. Trapped-ion standards have a potential frequency stability of 1 part in 10^{16} or better over long time periods. We consider the potential improvements this method could yield over conventional dynamical tests for unseen matter in the outer solar system possible now or anticipated in the near future.

Subject Headings: dark matter - solar system; general - gravitation] - relativity - radar astronomy

I. Introduction

An important unsettled issue concerning the nature of the solar system is whether there exist distributions of still unseen matter in the outer solar system. The possibilities include planetesimals remaining from the formation of the solar system, cometary belts, or even a tenth planet (for a review and certain limits, see Tremaine 1990). Unseen mass in the outer solar system could be detected by its dynamical effects on Uranus or Neptune, or on a deep space probe. However, present uncertainties in the data limit the sensitivity of these tests (Anderson and Standish 1986; Standish 1993). Because of the long orbital periods of Uranus and Neptune (84 years and 164.5 years, respectively), we may have to wait many more years before additional observations of their positions yield definite results. Matter at larger distances could still remain undetectable. The alternate method of tracking the motion of a deep space probe is limited by the effect of nongravitational forces, such as those produced by attitude control disturbances. For spin-stabilized spacecraft, such as Pioneer 10 and 11, acceleration models have been limited to an accuracy of 5×10^{-6} km/s². In this paper, we will consider a potential new method to test for unseen distributions of matter in the outer solar system.

In addition to the dynamical effects of a mass distribution, general relativity also predicts a gravitational frequency shift between two oscillators at different locations in a gravitational field. This effect can be measured in the solar system by transmitting a radio signal from a spacecraft equipped with a stable frequency standard (for a review, see Krisher 1990). However, because of the large distances involved in the outer solar system, several

months or even years may be required to probe the gravitational potential of an unknown source before a definite signature can be detected. We must therefore use a frequency standard which is highly stable over long time periods. The recent development of trapped-ion standards, which have this capability, makes such an experiment feasible now and potentially competitive with conventional dynamical tests.

The technological capabilities for such a test will be discussed further in the next Section. In Section III, the sensitivity of the test will be estimated for certain possible forms of unseen matter. We will consider the cases of 1) a spherical shell of either uniform density or with an isothermal density distribution, 2) a uniform, circular belt, and 3) a tenth planet. Concluding remarks appear in Section IV,

II. Technological Capabilities

Quartz crystal oscillators have been relied upon up to now for use in deep space probes. However, of the Pioneer and Voyager missions to the outer solar system, only the Voyager spacecraft were equipped with an ultrastable oscillator (USO). It was manufactured by Frequency Electronics Incorporated (FEI) in 1977. For 1000 second averaging intervals, the frequency stability is at 1 part in 10^{12} , although the random walk can be much larger. With this level of stability, it was possible to test the gravitational redshift due to Saturn with an accuracy of 1% from the Voyager 1 flyby in 1980 (Krisher, Anderson, Campbell 1990). Most recently, the Galileo mission has provided a test of the redshift due to the Sun at the same accuracy (Krisher, Morabito, and Anderson 1993; Morabito, Krisher, and Asmar 1993).

In principle, a similar type of experiment could be performed to test for a redshift due to unseen matter in the outer solar system. Presented in Fig. 1 is a plot of the Voyager 2 USO frequency from 1982 through the Neptune flyby in 1989. During this phase of the mission, the Voyager Radio Science Team collected data from the USO in order to monitor its health and performance. The data was generated by transmitting from the spacecraft a 2.3 GHz radio signal referenced to the USO to a tracking station of the NASA Deep Space Network, whose receiver was referenced to a hydrogen maser frequency standard. A coherently transponded signal referenced to the maser at the station was used to calibrate the motion of the spacecraft. We see that the most noticeable feature of the data is a linear decrease in the frequency at -11 mHz/day, which is consistent with expected aging effects

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in the USO itself. Unfortunately, the effects of aging and random walk are so large that they preclude a sensitive redshift test for unseen matter.

Aging effects are much less severe in an atomic frequency standard. For example, it is possible to keep the frequency drift of a hydrogen maser oscillator below 1 part in 10^{15} per day (Owings 1993). It has been proposed that a hydrogen maser standard be flown on a solar probe (Vessot 1991, 1993). At such a close proximity to a massive body, the maser would provide the capability to perform a highly precise gravitational redshift experiment (Krisher 1993). For a mission to the outer solar system, where smaller masses are involved, it is desirable to have still higher frequency stability. A cryogenic hydrogen maser theoretically could have much improved stability, but its actual capabilities remain to be determined (Walsworth *et al.* 1993).

There now exists, however, a new type of atomic frequency standard based upon the technique of storing ions in an electromagnetic trap (Vanier and Audoin 1989). For averaging times beyond 10^6 seconds, the stability of a trapped-ion standard has been demonstrated to exceed that of other atomic "clocks" (Allan, Weiss, and Peppler 1989). Mercury-ion standards employing a novel linear trap design, which has important advantages, are now being built and tested by a group at the Jet Propulsion Laboratory (Prestage *et al.* 1992; Tjoelker *et al.* 1993). Recent tests have demonstrated an Allan deviation of better than $10^{-13}/\sqrt{\tau}$ for averaging intervals τ . For $\tau > 10^6$ seconds, this level of performance results in a fractional frequency stability of better than 10^{-16} . With sufficient regulation of operating parameters and environmental perturbations, it may be possible to maintain this level of stability or greater over much longer time periods. Most importantly,

it is feasible to design a spacecraft standard satisfying the requirements of small size, small mass, and low power consumption (Maleki and Prestage 1994).

In order to exploit fully the stability of an atomic frequency standard, it is necessary to calibrate accurately the motion of the spacecraft and media effects on the radio signal. The motion of the spacecraft can be calibrated by measuring the Doppler shift of coherently transponded radio signals, in which the redshift effect cancel - out in the received signal. It is also possible, by various means, to calibrate the effects of interplanetary plasma and the Earth's ionosphere and troposphere. Accuracies have been achieved approaching the stability of the hydrogen maser used as a reference (1 part in 10^{15} ; Armstrong 1989). This accuracy could still be improved, however. An interesting possibility is to use a "four-link" radio system, which has been proposed in order to generate time correlated frequency data from which accurate media calibrations could be derived (Snarr *et al.* 1983; Vessot 1991, 1993). The plasma effects are dispersive and can be determined by incorporating multiple frequencies (Bertotti, Comoretto, and Less 1993).

111. Redshifts Due to Possible Matter Distributions

We will assume static distributions of matter in which there is no explicit dependence upon time. Then, according to general relativity, the frequency of a signal transmitted from a clock at rest at x_1 is related to the frequency received by a clock at rest at x_2 by the expression

$$f_2/f_1 = [g_{00}(x_1)/g_{00}(x_2)]^{1/2}, \quad (3.1)$$

The metric tensor $g_{\mu\nu}$ is specified by the Einstein equations for a given distribution of matter and appropriate boundary conditions. However, for weak gravitational fields $g_{00} = -1 + 2U/c^2$ to lowest order in the Newtonian potential U (defined positively), modulo possible constant factors depending upon boundary conditions. We will avoid these details by simply synchronizing our clocks appropriately. In order to determine the sensitivity which a redshift test could ultimately achieve, we will assume a fractional frequency stability of 10^{-17} .

a) Spherical Shell

First we will consider a uniform, spherically symmetric shell of radius r_s and having a vacant core of radius r_c . We could expect a Vacant core to encompass the inner solar system, because matter would have been swept away due to gravitational scattering by Jupiter and the inner planets. The Newtonian potential inside the shell is

$$U(r_c < r < r_1) = (2\pi G\rho/3)(3r_c^2 - r^2 - 2r_c^3 r^{-1}), \quad (3.2)$$

where ρ is the density. For a radio signal transmitted from inside the shell to a point inside the core, equation (3.1) predicts that

$$\frac{\Delta f}{f} = \frac{f_2 - f_1}{f_1} = \frac{2\pi G\rho}{3c^2}(r_1^2 + 2r_c^3 r_1^{-1} - 3r_c^2). \quad (3.3)$$

The amount of matter interior to r_1 is given by $M(r_1) = (4\pi\rho/3)(r_1^3 - r_c^3)$. If we define $\epsilon \equiv \Delta f/f$, then equation (3.3) provides the relation

$$M(r_1)/M_\odot = 2\epsilon \times 10^8 (r_1^3 - r_c^3)(r_1^2 + 2r_c^3 r_1^{-1} - 3r_c^2)^{-1}, \quad (3.4)$$

for distances in AU. Let us consider the case in which $r_1 = 30$ AU (the orbit of Neptune) and $r_c = 9.5$ AU (the orbit of Saturn). Then equation (3.4) yields the result $M(r_1) < 7.6 \times 10^9 |\epsilon| M_\odot$, or $M(r_1) < 10^{-7} M_\odot$ for $|\epsilon| < 10^{-17}$.

For an isothermal distribution of matter, the density varies with radius according to the expression

$$\rho(r) = \frac{\sigma^2}{2\pi G r^2}, \quad (3.5)$$

where σ is the mean velocity dispersion (Binney and Tremaine, 1987). In this case the Newtonian potential inside the shell is given by

$$U(r_c < r < r_s) \doteq 2\sigma^2[1 - r_c r^{-1} - \log(r/r_c)]. \quad (3.6)$$

(for an exact general relativistic solution, see Hojman, Pella, and Zamorano 1993). The mass at a radius r is $M(r) \doteq 2\sigma^2(r - r_c)$, from which we obtain the relation

$$\sigma^2 = 45.0 [M(r)/M_\odot] [\text{AU}/(r - r_c)] (\text{km/s})^2. \quad (3.7)$$

Reevaluating equation (3.3) for this case, we obtain for the redshift

$$\frac{\Delta f}{f} = 2(\sigma/c)^2 [1 - r_c r_1^{-1} - \log(r_c/r_1)], \quad (3.8)$$

which, with Eel. (3.7), yields the result

$$M(r_1)/M_\odot \doteq 10^8 \epsilon [(r_1 - r_c)/\text{AU}] [1 - r_c r_1^{-1} - \log(r_c/r_1)]^{-1}. \quad (3.9)$$

If we again consider a shell between the orbits of Saturn and Neptune, then $M(r_1) \doteq 1.1 \times 10^9 |\epsilon| M_\odot$. For $|\epsilon| < 10^{-7}$, we could thus detect $M(r_1) < 10^{-8} M_\odot$.

b) Uniform Belt

We will estimate the redshift effect of a uniform, circular belt situated at a heliocentric distance r_b by considering signals transmitted from near the

location of the belt. For this situation, the belt can be approximated by an infinitely long tube of radius r_t . The Newtonian potential at $r' < r_b$ is given by

$$U(r' < r_b) = - \left(\frac{GM}{\pi r_b} \right) \log \left(\frac{r_b - r}{r_t} \right), \quad (3.10)$$

where $M = (2\pi r_b)(\pi r_t^2)\rho$ is the total mass of the belt. Equation (3.10) implies that the redshift effect is of order $|\epsilon| \sim GM/c^2 \pi r_b$ for values of r consistent with our approximation. If we assume $r_b = 40$ AU, then $M \sim 10^{10} |\epsilon| M_\odot$. For $|\epsilon| = 10^{-1}$, we could thus detect a belt of mass $M \sim 10^{-7} M_\odot$.

c) Tenth Planet

The redshift effect due to a planetary body of mass M is simply given by

$$(\Delta f/f) = -(GM/c^2)(x_1^{-1} - x_2^{-1}), \quad (3.11)$$

to lowest order in the Newtonian potential, where x is the distance from the planet. Let us suppose that we begin measuring the redshift at 100 AU from the planet, and that the spacecraft passes within only $x_1 = 50$ AU. Equation (3.11) implies a mass sensitivity of $M = 10^{10} |\epsilon| M_\odot$, which yields $M < 10^{-7} M_\odot$ for $|\epsilon| < 10^{-17}$.

IV. Conclusions

From the preceding considerations, we conclude that a redshift test could have a detection sensitivity approaching $10^{-7} M_{\odot}$. In practice, the data would need to be fit to realistic models of possible matter distributions to determine the most likely source of any apparent signature. Unknown parameters in these models would involve the location or dimensions of the source, in addition to its mass. A rigorous covariance analysis should be done to determine precisely how well these parameters can be estimated in the presence of anticipated error sources, such as random walk of the frequency standards.

For the case of a spherical shell interior to Uranus or Neptune, considered recently by Anderson *et al.* (1989, 1994), a sensitivity of $10^{-7} M_{\odot}$, which equals $0.03 M_{\oplus}$, would provide about a factor of 2 improvement over the current sensitivity provided by analyzing the motion of Uranus. We could gain about a factor of 20 improvement over the sensitivity currently possible with Neptune. However, a redshift test could be most important for detecting matter beyond Neptune, for which planetary and spacecraft dynamical tests may be highly limited. This might verify the existence of the Kuiper Belt (Duncan, Quinn, and Tremaine 1988; Dyson 1992), for which candidate members have been detected recently at a heliocentric distance near 40 AU (Jewitt and Luu 1993). Tracking of Pioneer 10 has so far resulted in only a limit of $5 M_{\oplus}$ on the total mass of such a belt, (Anderson and Standish 1986). We also estimated that a tenth planet could be detected, even at large distances. A body with a mass of 1 to $5 M_{\oplus}$ has been proposed, in an orbit having a semimajor axis of 50 to 100 AU (Matese and Whitmire

1986; Barrington 1988). Present evidence for its existence is highly uncertain and plagued by differing interpretations of the available data on Uranus and Neptune (Seidelmann and Harrington 1988; Standish 1993).

Trapped-ion standards are being developed further at JPL. Tests of the long-term stability have been limited by the stability of the hydrogen maser standard used as a reference. The planned construction of several trapped-ion standards will permit intercomparisons to be performed independently of a reference of lesser stability (Tjoelker *et al.* 1993). Improvements are also possible in the linear trap design already in use (Prestage *et al.* 1993). In addition, the use of ytterbium ions, instead of mercury, is being investigated and could provide important advantages for the design of a spacecraft standard (Maleki and Prestage 1994). Although in this paper we focussed only on a single scientific goal, a spacecraft trapped-ion standard could provide for other interesting radio science experiments not discussed here.

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Figure Caption

Fig. 1 - Voyager 2 USO frequency in Hz versus time in clays. A bias of 2296481000 Hz has been subtracted from the data for convenience. The plot begins on 1 January 1982 and ends during 1990; for reference, the times of the Uranus and Neptune flybys are indicated by arrows.

